(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. II) EVEN THEORY EXAMINATION 2012-13

MATHEMATICS–II

Time : 3 Hours *Total Marks : 100*

SECTION–A

1. Attempt **all** parts of this question : **(2×10=20)**

(a) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 4y = 0$.

(b) Find the particular integral of $\frac{d^2y}{dx^2} = x^2 + 2x - 1$.

(c) Find the value of α and β for which $3x^2 = \alpha P_2(x) + \beta P_0(x)$.

(d) Determine the expression for $J_{-1/2}(x)$.

(e) Find the Laplace transform of $f(t) = t \sin \sqrt{7}\, t$.

(f) Find the function whose Laplace transform is

$$F(s) = \frac{8}{(s^2 - s - 2)}$$

(g) If $f(x) = 1$ is expanded in a Fourier sine series in $(0, \pi)$, then find the value of b_n.

(h) Classify the following differential equation in the first quadrant

$$y^2u_{xx} - x^2u_{yy} = 0$$

(i) Solve $\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$ using method of separation of variables.

(j) Write the boundary conditions and initial conditions for the displacement of a finite string of length L that is fixed at both ends and is released from rest with an initial displacement f(x).

SECTION–B

2. Attempt any **three** parts of the following : **(3×10=30)**

(a) Apply method of variation of parameters to find the general solution of

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 3x = \frac{e^t}{1+e^t}$$

(b) Find the Frobenius series solution of the following differential equation about x = 0.

$$2x^2y'' + 7x(x+1)y' - 3y = 0.$$

(c) Solve the following differential equation using Laplace Transform :

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = \sin x \; ; \; y(0) = 1, \; y'(0) = 0$$

(d) Find the Fourier series expansion of the following function of period 2π, defined as

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x < \pi \end{cases}$$

Hence evaluate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$

(e) Find the deflection u(x, t) of a tightly stretched vibrating string of unit length that is initially at rest and whose initial position is given by

$$\sin \pi x + \frac{1}{3}\sin(3\pi x) + \frac{1}{5}\sin(5\pi x), \; 0 \le x \le 1$$

SECTION–C

Note :– Attempt any **two** parts from each question of this Section.

(2×5)×5=50

3. (a) Solve $\frac{d^2y}{dx^2} + 4y = \sin^2 2x$ with conditions $y(0) = 0$, $y'(0) = 0$.

(b) Solve the following differential equation by reducing into normal form :

$$y'' + 2xy' + (x^2 - 8)y = x^2 e^{-\frac{1}{2}x^2}.$$

(c) Solve $\frac{dx}{dt} + 2x + 4y = 1 + 4t \; ; \; \frac{dy}{dt} + x - y = \frac{3}{2}t^2$.

4. (a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0, \; m \neq n$.

(b) Prove that $\frac{d}{dx}[xJ_n(x)\,J_{n+1}(x)] = x\{J_n^2(x) - J_{n+1}^2(x)\}$.

(c) Solve $4y'' + 9xy = 0$ in terms of Bessel function.

5. (a) Find the Laplace transform of $f(t) = \int_0^t \frac{1}{u}e^{-4u}\sin 3u \, du$.

(b) Find the inverse Laplace Transform of $\frac{1}{s^2(s+1)^2}$ using Convolution theorem.

(c) Find the Laplace transform of following function which is defined in one period as

$$f(t) = \begin{cases} t, & 0 \le t \le 1 \\ 1, & 1 \le t \le 2 \end{cases}$$

6. (a) Solve the following partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = \sin(2x + y)$$

(b) Solve $r + (a + b)s + abt = xy$.

(c) Find the half range cosine series expansion of $f(x) = x - x^2$ in $0 < x < 1$.

7. (a) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 0$ using method of separation of variables to obtain the solution that tends to zero as $y \to \infty$ for all x.

(b) Solve $\frac{\partial^2 u}{\partial x^2} = 2u + \frac{\partial u}{\partial y}$ using method of separation of variables subject to the conditions $u = 0$ and $\frac{\partial u}{\partial x} = e^{-3y}$ when $x = 0$ for all values of y.

(c) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = 0$, $u(x, 0) = 0$, $u(1, y) = 0$ and $u(x, 1) = 100 \sin \pi x$.